Engineering Notes

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The Determination of Load and Slope Transformation Matrices for Aeroelastic Analyses

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I. Introduction

THE idealization of aircraft structures usually requires different sets of grid locations for structural and aerodynamic analyses. In the aeroelastic analyses to represent the equilibrium of the forces, it has been convenient to define all the forces in the structural set. Therefore, there is a need to transform the load vectors from the aerodynamic set to the structural set. Likewise, it is also necessary to determine the streamwise slopes at the aerodynamic set in terms of the deformations of the structural set.

Several computational methods have been developed to perform such transformations. For example, the method discussed in Ref. 1 employs the least square technique to interpolate the displacements at the aerodynamic grids; whereas, the method of Ref. 2 employs the polynomial fit in-the-small. Alternatively, the methods of Refs. 3 and 4 utilize spline functions based on simple beam and infinite plate equations, respectively. In all these methods, the coefficients of interpolation functions need to be determined by solving a system of linear equations. For structures with irregularly spaced collocation points or with a large number of collocation points, the system of equations becomes ill-conditioned and results in inaccurate load and slope transformation matrices.

The present method employs the piecewise cubic monotone interpolation scheme of Ref. 5 to determine the displacements and slopes at the aerodynamic control points derived from the colums of the flexibility matrix. The procedure offers significantly increased accuracy, since the determination of the coefficients of the interpolation functions are not required in this method.

II. Transformation Matrices

It is assumed that the flexibility matrix F_{ss} is known for a structural set s, then the relation between the displacement vector q_s and the load vector R_s is given by

$$q_s = F_{ss} \cdot R_s \tag{1}$$

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The displacement vector q_a at the aerodynamic control points can be written as

$$q_a = F_{as} \cdot R_s \tag{2}$$

where F_{as} is to be determined from F_{ss}, using piecewise monotonic spline functions. Since the slopes can be computed from these spline functions, the slope vector α_a at the aerodynamic control points can be written as

$$\alpha_a = D_{as} \cdot R_s \tag{3}$$

where D_{as} are the slopes derived from the vectors of F_{as}.

Substituting for R_s from Eq. (1), the displacement transformation matrix T relating the displacements q_s and q_a is obtained to be

$$q_a = T \cdot q_s \tag{4}$$

where

$$T = F_{as} \cdot F_{ss}^{-1} \tag{5}$$

Similarly, the transformation matrix relating the vectors α_a and q_s is given by

$$\alpha_a = D \cdot q_s \tag{6}$$

where

$$D = D_{as} \cdot F_{ss}^{-1} \tag{7}$$

Let R_a be an aerodynamic load vector which is to be transformed to an equivalent load vector R_s. From the principle of virtual work, the relation between R_s and R_a can be derived as follows:

$$q_s^t \cdot R_s = q_a^t \cdot R_a \tag{8}$$

Substituting for q_a from Eq. (4), Eq. (8) can be rewritten as

$$q_s^t \cdot R_s = q_a^t \cdot T^t \cdot R_a \tag{9}$$

Hence

$$R_s = T^t \cdot R_a \tag{10}$$

Thus, the required load transformation matrix T^t and slope transformation matrix D are given by Eqs. (6) and (7), respectively.

III. Interpolation Procedure

This section describes the computational procedure employed to determine the matrices F_{as} and D_{as} from the known flexibility matrix F_{ss}. Each column of F_{ss} denotes the deformed shape of the structure due to a unit load at a structural node. It is then required to determine the displacement q_a at an aerodynamic grid point. The first step in the interpolation scheme is to select certain structural node points along the rib lines, say S_1-S_7; S_8-S_{13}, etc., as shown in Fig. 1. Interpolation functions along these rib lines will be established, using a modified version of Ref. 5.

This method employs a piecewise monotonic curve fitting technique using a parametric cubic function given by

$$P = Au^3 + Bu^2 + Cu + D \quad \text{for} \quad 0 \le u \le 1 \tag{11}$$

where P is any n dimensional coordinate (e.g., x, y, or z) and the coefficients are given by

$$\begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} P(0) \\ P(1) \\ \mathrm{d}P(0)/\mathrm{d}u \\ \mathrm{d}P(1)/\mathrm{d}u \end{Bmatrix} \tag{12}$$

in which $P(0)$ and $P(1)$ define the values of P at $u=0$ and $u=1$ and likewise $\mathrm{d}P(0)/\mathrm{d}u$ and $\mathrm{d}P(1)/\mathrm{d}u$ are the values of the parametric slopes at the end points.

The boundedness of the interpolated data is assured by examining the local parametric slopes and their monotonic behavior as described in Ref. 5.

To determine the displacements at the aerodynamic control points, it is necessary that a set of aerodynamic grids lying between two rib stations be selected. This can be performed automatically by the geometric condition that the sum of the vertex angles (α, β, γ, and δ) formed by joining the end points of the rib stations is 2π (Fig. 1). Once the aerodynamic sets are known, a second interpolation using the displacement at S_1, S_2, and S_3 (Fig. 2) computed from the previously established spline functions in the isoparametric coordinate system will determine the required displacements and the slopes at the aerodynamic control points. This procedure will be repeated for all aerodynamic points within the selected structural rib stations. Thus, the matrices F_{as} and D_{as} are determined from the known flexibility matrix F_{ss}.

The present method employs interpolations in two directions. The first interpolation establishes a spline function with a high degree of accuracy in displacements and slopes. The second interpolation is confined to the local interpolation between two or three rib stations. Since the spanwise slope is less important in the aerodynamic analysis, the first interpolation should be performed along the chord directions.

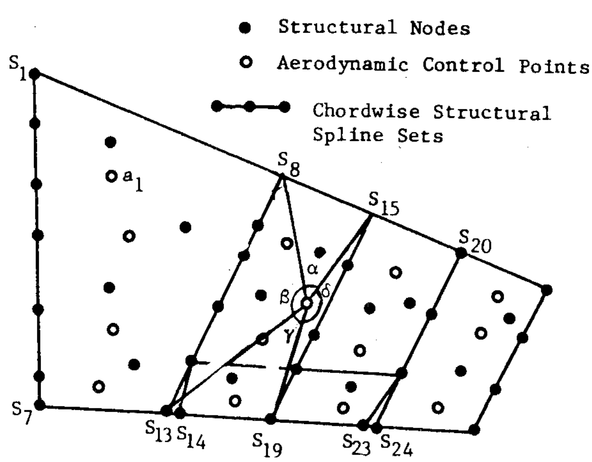

Fig. 1 A typical wing plan form showing the structural and aerodynamic control points.

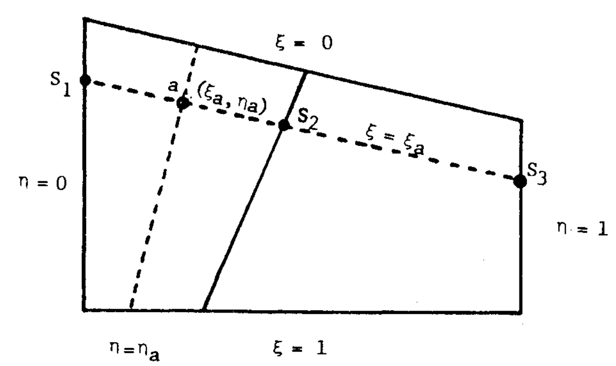

Fig. 2 Isoparametric interpolation scheme.

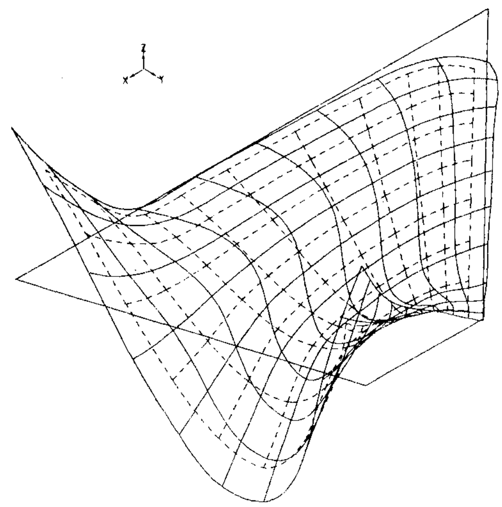

Fig. 3 Correlation of the original and the interpolated data of a vibration mode.

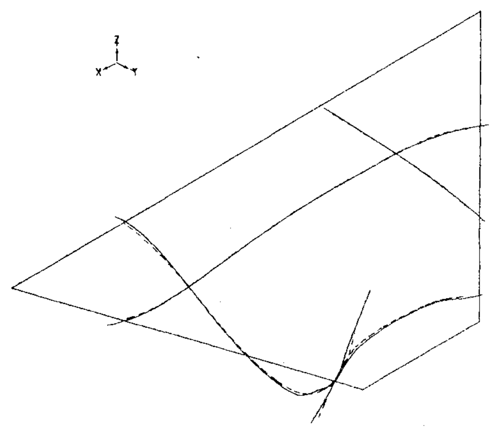

Fig. 4 A trimetric view of the chordwise and spanwise cuts of the superimposed surfaces.

IV. Results and Conclusions

To verify the computational accuracy of the proposed interpolation scheme, a low aspect ratio wing with 112 structural degrees of freedom of normal displacements was chosen. The aerodynamic set, in which the load and the slope transformation matrices were desired, consisted of 110 control points. For this configuration, the transformation matrices T and D were determined in the present method. However, this example using the method of Ref. 4 could not be performed due to an ill conditioned set of equations. The fifth vibration mode was transformed from the structural set to the aerodynamic set as given by Eq. (4). The original data and the transformed data are shown in Fig. 3. The solid lines denote the deformed shape of the vibration mode in the structural set, while the dotted lines denote the interpolated data at the aerodynamic grids, which are approximately at the center of the panels. For better understanding of the correlation, a trimetric view of two chordwise and two spanwise cuts is shown in Fig. 4. This clearly indicates a high degree of accuracy of the transformation matrix determined by the present method. Since there is no need to determine more than four spline coefficients at a time, the computational stability of this method is very well preserved. Furthermore, any complex configuration can be splined using a single patch which is not possible by other methods. Further it should be noted that the distribution of the aerodynamic loads on to the structural nodes, unlike in other methods, is dependent on the structural property. Hence, this procedure can be employed in the aeroelastic analysis for improved accuracy.

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